Frege, Frames, and Functional Concepts

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1. Frege on concepts, functions, and objects

1.1 Begriff und Gegenstand / Concept and object

(1) ‘The coat is blue’ = ‘the coat’ + ‘_____ is blue’

- ‘_____ is blue’ is the expression of a **concept** (**Begriff$_F$**).
  The expression is grammatically *incomplete* (ungesättigt, lit. 'unsaturated').
- The expression ‘the coat’ is an NP (**Name$_F$**). In appropriate contexts (there is exactly one object that meets the description), it denotes an **object** (**Gegenstand$_F$**).
- Combined with an NP, the expression of a concept yields a complete sentence.
- The sentence expresses that the concept is to be applied to the referent of the NP. Thereby, the sentence expresses a proposition (**Gedanke$_F$**).

1.1 Concept and object (ctd.)

(1) ‘The coat is blue’ = ‘the coat’ + ‘_____ is blue’

- A concept is incomplete. It is to be applied to an object (its argument).
- When a concept is applied to an object, it yields a **value**.
- A value is an object.
- The value of a concept is a **truth value**, TRUE of FALSE.
- A concept has an **extension** (**Begriffsumfang$_F$**), i.e. the set of all objects for which the value of he concept is TRUE
- Extensions are objects.
- Different concepts may have identical extensions,
  e.g. ‘___ is the morning star’ and ‘___ is the evening star’.

1. Frege on concepts, functions, and objects

1.2 Funktion und Argument / Function and argument

(2) a. $f(x) = 3 \cdot x^2 - x$
b. $3 \cdot \_\_^2 - \_\_$
c. $3 \cdot 8^2 - 8$

- ‘$3 \cdot \_\_^2 - \_\_$’, i.e. the form of the expression ‘$3 \cdot x^2 - x$’ with the variables removed, is expression of a **function** (**Funktion$_F$**).
- The expression ‘8’ is a term (Name$_F$). It denotes an object, the number 8..
- Insertion of a number term in the function expressions yields a number term.
- The expression ‘$3 \cdot 8^2 - 8$’ expresses that the function ‘$3 \cdot \_\_^2 - \_\_$’ is to be applied to the number 8. The expression is a term; it denotes the number 184.

1.2 Function and argument (ctd.)

(2) $f(x) = 3\cdot x^2 - x$

- A function is incomplete. It is to be applied to an object (its argument).
- When a function is applied to an object, it returns a **value**.
- A value is an object.
- A function has a **trajectory** (**Wertverlauf**).
  Frege's notation: $\dot{\varepsilon}(3\cdot\varepsilon^2 - \varepsilon)$
  modern: $\{ \langle x,y \rangle \mid y = 3\cdot x^2 - x \}$
- Trajectories are objects.
- Different functions may have identical trajectories,
  e.g. the functions indicated by '$3\cdot x^2 - x$' and '$x\cdot(3\cdot x - 1)$', respectively.

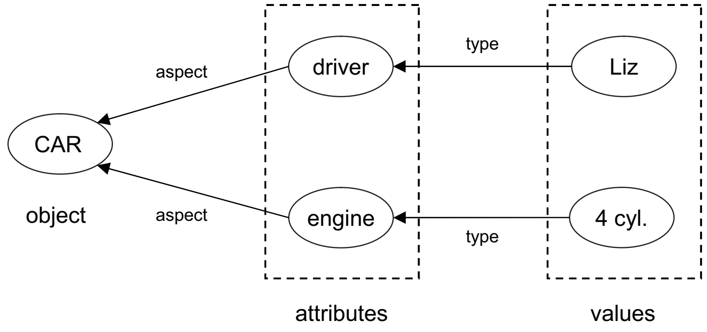
Concepts are functions that return truth values.

1.3 Three ontological levels

conceptual (intensional)	*extensional*	*single value*
function	trajectory	value
$3\cdot\_\_^2 - \_\_$	$\dot{\varepsilon}(3\cdot\varepsilon^2 - \varepsilon)$ $\{ \langle x, y \rangle \mid f(x) = y\}$ $f: A \longrightarrow B$	$3\cdot a^2 - a$ $f(a)$
concept	extension	a truth value
___ is blue	the blue objects $\{ x \mid x \text{ is blue}\}$ $P: A \longrightarrow 2$	[P(a)]

2. Frege's distinctions and Barsalou frames

2.1 Barsalou's frame graphs (from Barsalou 1992: 30, extract)



2.1 Barsalou's frame graphs (ctd.)

"By **concept** I mean the descriptive information that people represent cognitively for a category, […] " (Barsalou 1992:31)

"An **attribute** is a concept that describes an aspect of at least some category members." (Barsalou 1992:30)

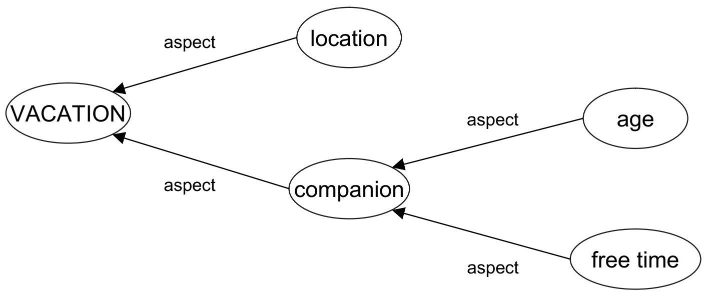
"A concept is only an attribute if it describes an aspect of a larger whole. When people consider *color* in isolation (e.g., thinking about their favorite color), it is not an attribute but is simply a concept." (Barsalou 1992:30)

"**Values** are subordinate concepts of an attribute. [...] they inherit information from their respective attribute concepts. […] Values inherit the extrinsic fact that they are an aspect of category members. Because *engine* is an aspect of *car*, its values are an aspect of *car* as well." (Barsalou 1992:31)

Are values attributes?

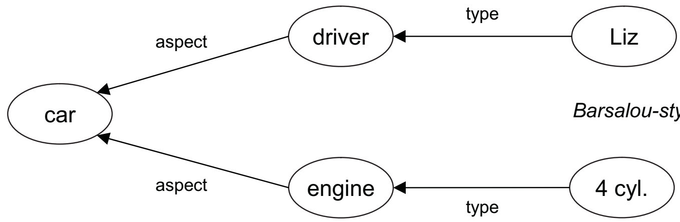
2.1 Barsalou's frame graphs

(from Barsalou 1992: 30, extract)

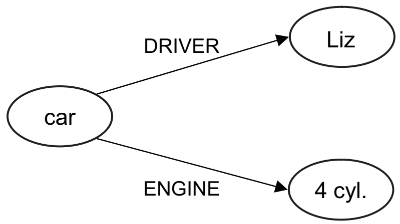


Note: Actually, AGE is not an attribute of the attribute COMPANION (the attribute is not old or young), but an attribute of the *value* of the attribute COMPANION

The car frame in Fregean description

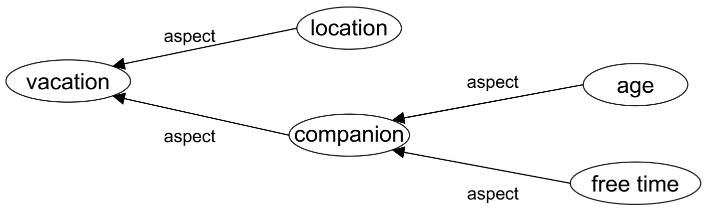


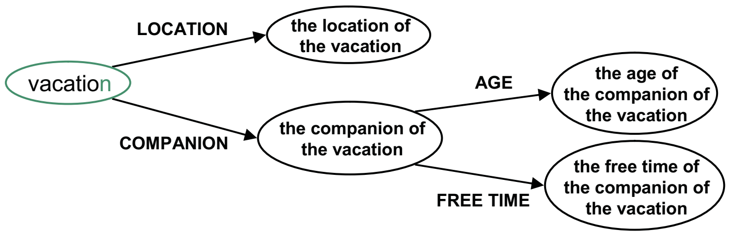
Barsalou-style original



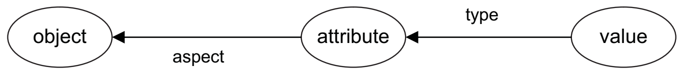
Fregean adaptation

The vacation frame in Fregean description





The basic frame unit according to Barsalou

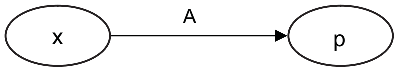


The attribute is an aspect of the object described;
its value is a (sub-)type of the attribute [as a superordinate concept].

The basic frame unit in Fregean fashion

The attribute is a function which returns a value for the object described;
the value specification is type information about the value.

The basic frame unit in Fregean fashion (2)



The basic unit is to be interpreted as follows:

p(A(x)) , i.e. the value of the attribute A for the argument x is of type p.

‘the engine of the car is of the type four-cylinders engine’

If the type information of the value is a precise (atomic) type t, the unit reads as:
‘the driver of the car is [of the type] Liz’

A(x) = t.

3. Attributes

3.1 Attributes and types

- Being a function, an attribute has a *domain* and a *range*.

  The domain is the set of all objects for which the attribute returns a value.

  The range is the set of all objects which are attribute values for some argument.

- The domain and the range constitute *types* in a type hierarchy.
  Types are extensions of (sortal) concepts.

  Value specifications specify a sub-type of the range of an attribute.

3.1 Attributes and types

- Definition:
  An attribute is **natural**, iff its domain is maximal.

  Note that an attribute is defined conceptually; for every argument in its domain, the same rule is to be applied for determining its value.

  Example: The attribute COLOUR is a maximal attribute with the domain of all visible objects, but not with the domain of wine, human hair or human eyes.

- Definition:
  A type is **maximal**, iff ???
  — intuitively: if it comprises only sub-types of the same kind,
  not e.g. colours along with shapes, prices, temperatures, and ages

- There is a correspondence between natural attributes and natural types:
  The range of a natural attribute is a natural type, and
  natural types are the range of natural atributes.

3.1 Attributes and types

Barsalou takes advantage of the dualism between attributes and their range types.

If an attribute is identified with the corresponding type / concept of its range

- the tandem of attribute and corresponding type can play the double role of an attribute that “describes an aspect of a larger whole” and as “a concept” when “consider[ed] […] in isolation”. (cf. Barsalou quotes above)

- value (specification)s can be considered “subordinate concepts of an attribute”, namely of the attribute range concept. (cf. Barsalou quotes above)

- attributes can be considered having attributes of their own: identified with their value ranges, the sub-attribute is a subordinate concept of the super-attribute.

3.2 Attributes and types from the Fregean perspective

Attributes are functions, i.e. something conceptual.

Every attribute is naturally associated with two (unary) concepts,

- the **domain concept** D(A), expressed by
  '___ has a value for the attribute A'
- the **range concept** R(A), expressed by
  ' ___ is a value of the attribute A for some argument'

3.3 Sortal concepts and functional concepts: logical properties

Barsalou's frames are frames for **sortal concepts** (i.e. 'concepts' in the sense of Guarino 1992). Sortal concepts correspond to one-place first order predicates, or one-place Fregean 'concepts'.

- When applied to an argument, they yield a truth value w.
  They may be true for an open number of objects.

Functions are **functional concepts**. They correspond to first order function terms. They differ from sortal concepts:

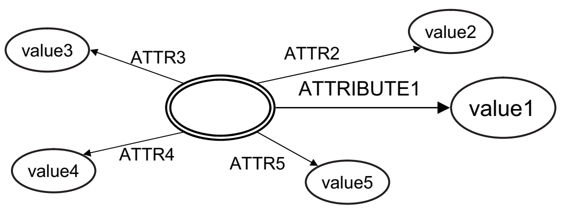
- When applied to an appropriate argument, they result in a conceptually *unique* description of an object; the object is not a truth value.

Taken as a description of objects,

- functional concepts (when applied to an argument) are unique descriptions, sortal concepts are not
- for functional concepts, the description depends on the choice of argument(s), for sortal concepts, the description is independent.

3.4 Sortal concepts and functional concepts: frame structures

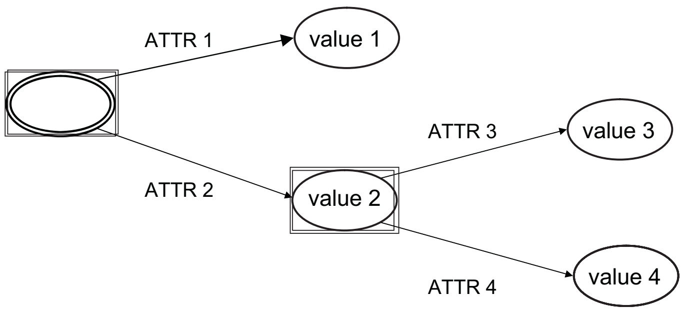
Essential structure of a **sortal concept**:
(the double-lined node marks the *referent*, i.e. the object of description. More attributes may be added recursively to nodes of the structure)



Essential structure of a (one-place) functional / attribute concept:
(the rectangle marks an open argument)

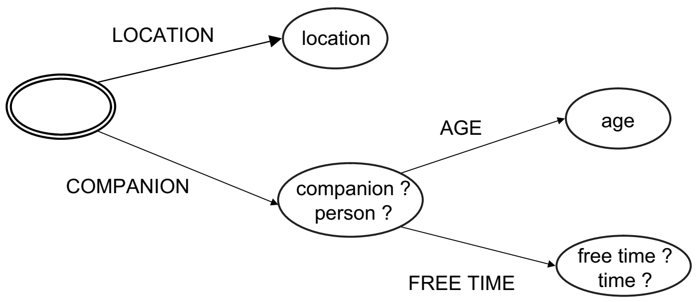
3.5 Sortal concepts and functional concepts: frames structure

A Fregean frame is a sortal concept composed of functional concepts recursively applied to the referent of the frame:



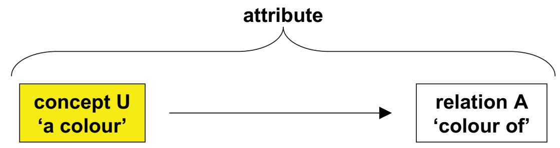
3.6 Attributes and types

The duality of functional attributes and their range types virtually doubles the labels in a frame graph with general value types:

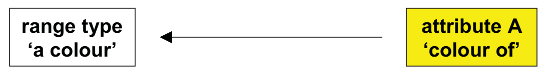


3.7 Attributes and concepts: Guarino 1992

Attributes are unary relations (i.e. 'concepts') U associated with a binary relation, their 'relational interpretation' R, such that if R(x,y) then U(y); the concept is primary:



Alternatively:
An attribute is a function (i.e. a special type of binary relation). As such it is associated with its range and the range type; the attribute is primary.



3.8 Attributes and concepts: Which is prior?

"Concepts are prior"

- "Unary relations are simpler than binary relations."
- Most nouns denote unary concepts.
- Unary concepts may be implicitly relational; cf. Guarino's "founded concepts":

  for the member of a founded concept, necessarily exists a member of a different concept [to which it is related, S.L.]
  e.g. 'father', 'age' (but not 'head')

"Attributes are prior"

- Attributes are conceptually less complex than unary concepts.
- If Barsalou is right, unary concepts are composed of attributes.
- An attribute naturally determines the corresponding range type concept.
- Deriving an attribute from a range type concept is conceptually impossible, except …..

3.8 Attributes and concepts: Which is prior?

… except for "founded" concepts?

ex. 'father'

$\forall x(x \in father \rightarrow \exists y(y \in person \wedge \neg x \geq y \wedge \neg y \geq x))$

There is no way to derive the binary attribute/role from this condition:

$f(y) = x$ iff_{def} $x \in father \wedge y \in person \wedge \neg x \geq y \wedge \neg y \geq x \wedge ??$

In order to achieve the correct assignments, one would need an assignment of children to their father: 'child(x)'.

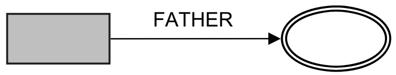
$f(y) = x$ iff_{def} $x \in father \wedge y \in child(x)$

'child' is binary; it is just the inverse of 'father'/'mother'; the definition is circular.

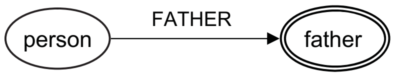
In general, a binary relation cannot be defined in terms of unary conditions.

3.8 Attributes and concepts: Which is prior?

Frame representation of the function/attribute FATHER



Frame representation of the unary concept ‘(a) father’ – crucially involves the attribute



4. Attribute terms in language

4.1 Three uses of attribute nouns — and Frege, again

(3) ‘The colour of the coat is a colour.’

- ‘the colour of the coat’ + ‘____ is a colour’
  argument expression — concept expression
  *for the **range concept of the attribute COLOUR***

- ‘the colour of the coat’
  *description of the **value of the attribute COLOUR for the coat***

- ‘the colour of ____’ + ‘the coat’
  function expression — argument expression
  *for the **attribute function***

4.1 Three uses — and Frege, again

(3) ‘The colour of the coat is a colour.’

- ‘the colour of the coat’
  Here, the noun *colour* is used as a **functional noun**
  i.e. as a noun with a possessor argument (‘of the coat’),
  and inherently unique (definite article)

- ‘___ is a colour’
  Here, the noun *colour* is used as a **sortal noun**
  i.e. as a noun without a possessor argument,
  and not inherently unique (indefinite article)

A systematic type shift — the **Guarino shift** — maps the attribute concept on the sortal concept of its range type.

4.2 Three types of predications about attributes

(1) “extensional” *The colour of the coat <u>is red</u>.*
The temperature of the cooling water <u>is 95°C</u>.
The father of Giancarlo <u>is from Chicago</u>.

The subject NP names the value of an attribute for a particular argument;
The VP predicates (and thereby specifies) the value [for the given context].
This is a predication about a single value of the attribute.

(2) “intensional” *The temperature of the cooling water <u>is rising</u>.*
The engine of the car <u>was replaced</u>.
The wife of Giancarlo <u>has changed</u>.

The subject NP names the value of an attribute for a particular argument.
The VP makes an predication about a change of the value of he attribute.
The argument of the predication is an object – the trajectory of the attribute.

Intensionality test:
The NP cannot be replaced salva veritate by an NP with the same referent.

4.2 Three types of predications about attributes

(3) "concealed question"
The tag at the bag *displays* *the price* (= *what the price* / *how much it costs*).
He does not *know* *his father* (= *who his father is*).

The argument NP names an attribute. The predication presupposes that there are several values possible. The predication is intensional.
For example: if the price of the bag happens to be the same as the price of the shoes, we cannot infer from the first sentence that

The tag at the bag displays the price of the shoes.

Thus: These predications are not about the given value of the attribute.
Are they about the trajectory?

5 Conclusions

- Applying Frege's ontological categories to concept representations in ontologies or frames helps to disentangle
  (a) the conceptual level from the object level
  (b) function[al concept]s from [sortal] concepts
- Combining Frege's ontology with Barsalou's theory of cognition contributes
  to the understanding of the basic architecture of conceptual representations:
  the interdependence of functional attributes and sortal concepts;
- … shows that concept representation is irreducibly based on relational, if not functional, concepts — even the representation of non-relational concepts.

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