1. Frege on concepts, functions, and objects

1.1 Begriff und Gegenstand / Concept and object

(1) ‘The coat is blue’ = ‘the coat’ + ‘_____ is blue’
   - ‘_____ is blue’ is the expression of a concept (Begriff).
   - The expression is grammatically incomplete (ungesättigt, lit. ‘unsaturated’).
   - The expression ‘the coat’ is an NP (Name). In appropriate contexts (there is exactly one object that meets the description), it denotes an object (Gegenstand).
   - Combined with an NP, the expression of a concept yields a complete sentence.
   - The sentence expresses that the concept is to be applied to the referent of the NP. Thereby, the sentence expresses a proposition (Gedanke).

1.1 Concept and object (ctd.)

- A concept is incomplete. It is to be applied to an object (its argument).
- When a concept is applied to an object, it yields a value.
- A value is an object.
- The value of a concept is a truth value, TRUE or FALSE.
- A concept has an extension (Begriffsumfang), i.e. the set of all objects for which the value of the concept is TRUE.
- Extensions are objects.
- Different concepts may have identical extensions,
  e.g. ‘___ is the morning star’ and ‘___ is the evening star’.

1.2 Funktion und Argument / Function and argument

(2) a. \( f(x) = 3 \cdot x^2 - x \)
   b. \( 3 \cdot \_ \_^2 - \_ \_ \)
   c. \( 3 \cdot 8^2 - 8 \)
   - ‘\( 3 \cdot \_ \_^2 - \_ \_ \)’, i.e. the form of the expression ‘\(3 \cdot x^2 - x\)’ with the variables removed, is expression of a function (Funktion).
   - The expression ‘\(8\)’ is a term (Name). It denotes an object, the number 8.
   - Insertion of a number term in the function expressions yields a number term.
   - The expression ‘\(3 \cdot 8^2 - 8\)’ expresses that the function ‘\(3 \cdot \_ \_^2 - \_ \_ \)’ is to be applied to the number 8. The expression is a term; it denotes the number 184.
1.2 Function and argument (ctd.)

(2) \( f(x) = 3 \cdot x^2 - x \)

- A function is incomplete. It is to be applied to an object (its argument).
- When a function is applied to an object, it returns a value.
- A value is an object.
- A function has a trajectory (Wertverlauf).
  Frege’s notation: \( \text{ἐ} (3 \cdot \text{ε}^2 - \text{ε}) \)
  modern: \( \{ (x, y) \mid y = 3 \cdot x^2 - x \} \)
- Trajectories are objects.
- Different functions may have identical trajectories, e.g. the functions indicated by \( 3 \cdot x^2 - x \) and \( x \cdot (3 \cdot x - 1) \), respectively.

Concepts are functions that return truth values.

1.3 Three ontological levels

<table>
<thead>
<tr>
<th>conceptual (intensional)</th>
<th>extension</th>
<th>single value</th>
</tr>
</thead>
<tbody>
<tr>
<td>function</td>
<td>trajectory</td>
<td>value</td>
</tr>
<tr>
<td>( 3 \cdot x^2 - x )</td>
<td>( \text{ἐ}(3 \cdot \text{ε}^2 - \text{ε}) )</td>
<td>( 3 \cdot a^2 - a )</td>
</tr>
<tr>
<td>( { (x, y) \mid f(x) = y } )</td>
<td>( f: A \rightarrow B )</td>
<td>( f(a) )</td>
</tr>
<tr>
<td>concept</td>
<td>extension</td>
<td>a truth value</td>
</tr>
<tr>
<td>___ is blue</td>
<td>the blue objects</td>
<td>( [P(a)] )</td>
</tr>
<tr>
<td>( { x \mid x \text{ is blue} } )</td>
<td>( P: A \rightarrow 2 )</td>
<td></td>
</tr>
</tbody>
</table>

2. Frege’s distinctions and Barsalou frames

2.1 Barsalou’s frame graphs (ctd.)

“By concept I mean the descriptive information that people represent cognitively for a category, […]” (Barsalou 1992:31)

“An attribute is a concept that describes an aspect of at least some category members.” (Barsalou 1992:30)

“A concept is only an attribute if it describes an aspect of a larger whole. When people consider color in isolation (e.g., thinking about their favorite color), it is not an attribute but is simply a concept.” (Barsalou 1992:30)

“Values are subordinate concepts of an attribute. […] they inherit information from their respective attribute concepts. […] Values inherit the extrinsic fact that they are an aspect of category members. Because engine is an aspect of car, its values are an aspect of car as well.” (Barsalou 1992:31)

Are values attributes?
2.1 Barsalou’s frame graphs
(from Barsalou 1992: 30, extract)

Note: Actually, AGE is not an attribute of the attribute COMPANION (the attribute is not old or young), but an attribute of the *value* of the attribute COMPANION.

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The car frame in Fregean description

Barsalou-style original

Fregean adaptation

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The vacation frame in Fregean description

The basic frame unit according to Barsalou

The attribute is an aspect of the object described; its value is a (sub-)type of the attribute [as a superordinate concept].

The basic frame unit in Fregean fashion

The attribute is a function which returns a value for the object described; the value specification is type information about the value.
The basic frame unit in Fregean fashion (2)

\[ x \xrightarrow{A} p \]

The basic unit is to be interpreted as follows:

\[ p(A(x)) \], i.e. the value of the attribute \( A \) for the argument \( x \) is of type \( p \).

‘the engine of the car is of the type four-cylinders engine’

If the type information of the value is a precise (atomic) type \( t \), the unit reads as:

‘the driver of the car is [of the type] Liz’

\[ A(x) = t. \]

3. Attributes

3.1 Attributes and types

- Definition:
  An attribute is natural, iff its domain is maximal.
  \textit{Note that an attribute is defined conceptually; for every argument in its domain, the same rule is to be applied for determining its value.}
  
  Example: The attribute COLOUR is a maximal attribute with the domain of all visible objects, but not with the domain of wine, human hair or human eyes.

- Definition:
  A type is maximal, iff ???
  — intuitively: if it comprises only sub-types of the same kind,
  not e.g. colours along with shapes, prices, temperatures, and ages

- There is a correspondence between natural attributes and natural types:
  \textit{The range of a natural attribute is a natural type, and natural types are the range of natural attributes.}

Barsalou takes advantage of the dualism between attributes and their range types.

If an attribute is identified with the corresponding type / concept of its range

- the tandem of attribute and corresponding type can play the double role of an attribute that “describes an aspect of a larger whole” and as “a concept” when “consider[ed] […] in isolation”. (cf. Barsalou quotes above)

- value (specification)s can be considered “subordinate concepts of an attribute”, namely of the attribute range concept. (cf. Barsalou quotes above)

- attributes can be considered having attributes of their own: identified with their value ranges, the sub-attribute is a subordinate concept of the super-attribute.
3.2 Attributes and types from the Fregean perspective

Attributes are functions, i.e. something conceptual.

Every attribute is naturally associated with two (unary) concepts,

- the domain concept \( D(A) \), expressed by
  
  ‘\_\_\_ has a value for the attribute A’

- the range concept \( R(A) \), expressed by
  
  ‘\_\_\_ is a value of the attribute A for some argument’

3.3 Sortal concepts and functional concepts: logical properties

Barsalou’s frames are frames for sortal concepts (i.e. ‘concepts’ in the sense of Guarino 1992). Sortal concepts correspond to one-place first order predicates, or one-place Fregean ‘concepts’.

- When applied to an argument, they yield a truth value \( w \). They may be true for an open number of objects.

Functions are functional concepts. They correspond to first order function terms. They differ from sortal concepts:

- When applied to an appropriate argument, they result in a conceptually unique description of an object; the object is not a truth value.

Taken as a description of objects,

- functional concepts (when applied to an argument) are unique descriptions, sortal concepts are not
- for functional concepts, the description depends on the choice of argument(s), for sortal concepts, the description is independent.

3.4 Sortal concepts and functional concepts: frame structures

Essential structure of a sortal concept:

(the double-lined node marks the referent, i.e. the object of description. More attributes may be added recursively to nodes of the structure)

Essential structure of a (one-place) functional / attribute concept:

(the rectangle marks an open argument)

3.5 Sortal concepts and functional concepts: frames structure

A Fregean frame is a sortal concept composed of functional concepts recursively applied to the referent of the frame:
3.6 Attributes and types
The duality of functional attributes and their range types virtually doubles the labels in a frame graph with general value types:

```
LOCATION
  ↓
  location

AGE
  ↓
  age

COMPANION
  ↓
  companion
    ?
  person

FREE TIME
  ↓
  free time
    ?
  time
```

3.7 Attributes and concepts: Guarino 1992
Attributes are unary relations (i.e. ‘concepts’) U associated with a binary relation, their ‘relational interpretation’ R, such that if R(x,y) then U(y); the concept is primary:

Alternatively:
An attribute is a function (i.e. a special type of binary relation). As such it is associated with its range and the range type; the attribute is primary.

3.8 Attributes and concepts: Which is prior?
“Concepts are prior”
- “Unary relations are simpler than binary relations.”
- Most nouns denote unary concepts.
- Unary concepts may be implicitly relational; cf. Guarino’s “founded concepts”:
  for the member of a founded concept, necessarily exists a member of a different concept [to which it is related, S.L.]
  e.g. ‘father’, ‘age’ (but not ‘head’)

“Attributes are prior”
- Attributes are conceptually less complex than unary concepts.
- If Barsalou is right, unary concepts are composed of attributes.
- An attribute naturally determines the corresponding range type concept.
- Deriving an attribute from a range type concept is conceptually impossible, except ….

“… except for ‘founded’ concepts?”
ex. ‘father’

\[
\forall x (x \in \text{father} \rightarrow \exists y (y \in \text{person} \land \lnot xy \land \lnot yx))
\]

There is no way to derive the binary attribute/role from this condition:

\[
f(y) = x \text{ iff } \text{def } x \in \text{father} \land y \in \text{person} \land \lnot xy \land \lnot yx \land ?\]

In order to achieve the correct assignments, one would need an assignment of children to their father: ‘child(x)’.

\[
f(y) = x \text{ iff } \text{def } x \in \text{father} \land y \in \text{child}(x)
\]

‘child’ is binary; it is just the inverse of ‘father’/‘mother’; the definition is circular.

In general, a binary relation cannot be defined in terms of unary conditions.
3.8 Attributes and concepts: Which is prior?
Frame representation of the function/attribute FATHER

Frame representation of the unary concept '(a) father' – crucially involves the attribute

4. Attribute terms in language

4.1 Three uses of attribute nouns — and Frege, again

(3) ‘The colour of the coat is a colour.’
- ‘the colour of the coat’ + ‘___ is a colour’
  - argument expression concept expression for the range concept of the attribute COLOUR

- ‘the colour of the coat’
  - description of the value of the attribute COLOUR for the coat

- ‘the colour of ___’ + ‘the coat’
  - function expression argument expression for the attribute function

4.2 Three types of predications about attributes

(1) ‘extensional’ The colour of the coat is red.
The temperature of the cooling water is 95°C.
The father of Giancarlo is from Chicago.

The subject NP names the value of an attribute for a particular argument;
The VP predicates (and thereby specifies) the value [for the given context].
This is a predication about a single value of the attribute.

(2) ‘intensional’ The temperature of the cooling water is rising.
The engine of the car was replaced.
The wife of Giancarlo has changed.

The subject NP names the value of an attribute for a particular argument.
The VP makes an predication about a change of the value of he attribute.
The argument of the predication is an object – the trajectory of the attribute.

Intensionality test:
The NP cannot be replaced salva veritate by an NP with the same referent.
4.2 Three types of predications about attributes

(3) “concealed question”

The tag at the bag displays the price (= what the price / how much it costs).
He does not know his father (= who his father is).

The argument NP names an attribute. The predication presupposes that there are several values possible. The predication is intensional.
For example: if the price of the bag happens to be the same as the price of the shoes, we cannot infer from the first sentence that

The tag at the bag displays the price of the shoes.

Thus: These predications are not about the given value of the attribute. Are they about the trajectory?

5 Conclusions

- Applying Frege’s ontological categories to concept representations in ontologies or frames helps to disentangle
  (a) the conceptual level from the object level
  (b) functional concept[s] from [sortal] concepts

- Combining Frege’s ontology with Barsalou’s theory of cognition contributes
to the understanding of the basic architecture of conceptual representations;
the interdependence of functional attributes and sortal concepts;

- … shows that concept representation is irreducibly based on relational, if not functional, concepts — even the representation of non-relational concepts.

Selected references


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